Assignment 9 – MA 542

Due in class: Wednesday, Apr. 7, 2010

Chapter 20: 30, 32, 34 (Edition 6: same numbers)

35) Let F, K, and L be fields with $F \subseteq K \subseteq L$. If L is a splitting field for some nonconstant polynomial $f(x)$ over F, show that L is a splitting field for $f(x)$ over K.

(1) Show $\pi^2 - 1$ is algebraic over $\mathbf{Q}(\pi^3)$.

(2) Let $D = (\mathbf{Z}/p\mathbf{Z})[T]$ and let $f(x) = \sum_{i=0}^{n} a_i x^i \in D[x]$. Suppose $T \nmid a_n$, $T \mid a_i$ for $i < n$, but $T^2 \nmid a_0$. Show that $f(x)$ is irreducible over $F = \mathrm{Frac}(D) = (\mathbf{Z}/p\mathbf{Z})(T)$.

(3) Use the previous exercise to produce infinitely many irreducible inseparable polynomials over $F = (\mathbf{Z}/p\mathbf{Z})(T)$.

(4) Find the minimal polynomial of α over F in the following situations:

(a) $\alpha = e^{2\pi i/3}$, $F = \mathbf{Q}$

(b) $\alpha = i\sqrt{2}$, $F = \mathbf{Q}(\sqrt{2})$

(c) $\alpha = i\sqrt{2}$, $F = \mathbf{Q}$

(d) $\alpha = a + 1 \in F(a)$, $F = \mathbf{Z}/2\mathbf{Z}$, where $F(a)$ is the field described in exercise 8 of chapter 20, i.e. a is a root of $x^3 + x + 1$.