Math 221 – Limit laws with infinite limits

Fall 2011

Below is a tabular display of the limit laws when infinity is involved. Perhaps the most important thing to notice about this table is all the **MW!**s. These represent cases where the limit law doesn't tell you anything, and you need to do ... More Work! In such cases, you need to either factor, rearrange the limit, or use later techniques (such as l'Hospital's Rule).

In the table, a is a real number or $\pm\infty$, b is a real number, and $f(x)$ and $g(x)$ are two functions whose limit at a is given by the first and second columns, respectively. The third through sixth columns tell you what the limit laws say about the limit as x approaches a of the function in the first row. A '$\begin{smallmatrix}\text{bit}\\ \text{mw}\end{smallmatrix}$' means that the limit law tells you the limit does not exist, but it may be $\pm\infty$. In this case, you need to do a bit more work, as we saw in class. Note that you can replace the limits with limits from the left or right.

$\lim_{x\to a} f(x)$	$\lim_{x\to a} g(x)$	$f+g$	$f-g$	fg	f/g
∞	∞	∞	**MW!**	∞	**MW!**
	$-\infty$	**MW!**	∞	$-\infty$	**MW!**
	0	∞	∞	**MW!**	$\begin{smallmatrix}\text{bit}\\ \text{mw}\end{smallmatrix}$
	$\pm b$, (with $b>0$)	∞	∞	$\pm\infty$	$\pm\infty$
$-\infty$	∞	**MW!**	$-\infty$	$-\infty$	**MW!**
	$-\infty$	$-\infty$	**MW!**	∞	**MW!**
	0	$-\infty$	$-\infty$	**MW!**	$\begin{smallmatrix}\text{bit}\\ \text{mw}\end{smallmatrix}$
	$\pm b$, (with $b>0$)	$-\infty$	$-\infty$	$\mp\infty$	$\mp\infty$
0	∞	∞	$-\infty$	**MW!**	0
	$-\infty$	$-\infty$	∞	**MW!**	0
	0	0	0	0	**MW!**
	$\pm b$, (with $b>0$)	$\pm b$	$\mp b$	0	0
$\pm b$, (with $b>0$)	∞	∞	$-\infty$	$\pm\infty$	0
	$-\infty$	$-\infty$	∞	$\mp\infty$	0
	0	$\pm b$	$\pm b$	0	$\begin{smallmatrix}\text{bit}\\ \text{mw}\end{smallmatrix}$