Math 243

Spring 2017

Asst 10

Selected Solutions

## §12.5:

(2) $a_T$: Step 1: Find $\vec{v}(t)$:

$v(t) = (3, 1, -3)$

Step 2: $a_T = \frac{d}{dt}|\vec{v}|$

$|\vec{v}(t)| = \text{const}$

so $\boxed{a_T = 0}$

$a_N$: Step 1: Find $\vec{a}(t)$

$\vec{a}(t) = \vec{v}\,'(t) = 0$

Step 2: $a_N = \sqrt{|\vec{a}|^2 - a_T^2}$

$\boxed{a_N = 0}$

(4) $a_T$: Step 1: Find $\vec{v}(t)$

$\vec{v}(t) = (t\cdot(-\sin t) + \cos t,\ t\cdot\cos(t) + \sin(t),\ 2t)$

Step 2: $a_T = \frac{d}{dt}|\vec{v}(t)|$

$= \frac{d}{dt}\left(\sqrt{t^2\sin^2(t) \cancel{-2t\sin t\cos t} + \cos^2 t + t^2\cos^2(t) \cancel{+2t\sin(t)\cos(t)} + \sin^2(t) + 4t^2}\right)$

$= \frac{d}{dt}\left(\sqrt{t^2 + 1 + 4t^2}\right) = \frac{d}{dt}\left(\sqrt{5t^2+1}\right) = \frac{1}{2\sqrt{5t^2+1}}\cdot 10t$

$= \frac{5t}{2\sqrt{5t^2+1}}$

at $t=0$: $\boxed{a_T = 0}$

(9) (continued) $a_N$: Step 1: Find $\vec{a}(t)$

$$\vec{a}(t) = \vec{v}\,'(t) = \left(-t\cos t - \sin t - \sin(t),\ -t\sin(t) + \cos t + \cos(t),\ 2\right)$$

Step 2: at $t=0$:

$$a_N = \sqrt{|\vec{a}(0)| - a_T(0)}$$

now $a_T(0) = 0$

& $\vec{a}(0) = (0, 2, 2)$

so $a_N = \sqrt{2^2 + 2^2} = \boxed{\sqrt{8}}$

(10) $\vec{B} = \vec{T} \times \vec{N}$ $\qquad \vec{T} = (\cos(t), \sin(t), 0)$

$\qquad \vec{N} = (-\sin(t), \cos(t), 0)$

so $$\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \cos(t) & \sin(t) & 0 \\ -\sin(t) & \cos(t) & 0 \end{vmatrix} = \left(0, 0, \cos^2(t) + \sin^2(t)\right)$$

$$= \boxed{(0, 0, 1)}$$

$$= \hat{k}$$

$\tau$: Note that $\vec{B}$ is constant so $\frac{d\vec{B}}{ds} = 0$

so $\tau = -\frac{d\vec{B}}{ds} \cdot \vec{N} = \boxed{0}$

§12.6:

(2) $\vec{v} = \dot{r}\,\vec{u}_r + r\dot{\theta}\,\vec{u}_\theta$ $\qquad \dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt}$

$$= \frac{d}{d\theta}\left(a\sin(2\theta)\right) \cdot 2t$$

$$= a\cos(2\theta) \cdot 2 \cdot 2t$$

$$= 4t\,a\cos(2\theta)$$

so $\boxed{\vec{v} = (4at\cos(2\theta))\,\vec{u}_r + 2at\sin(2\theta)\,\vec{u}_\theta}$

(2) (cont'd)

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\vec{u}_\theta$$

$$\dot{r} = 4t\,a\cos 2\theta$$

$$\text{so } \ddot{r} = \frac{d}{dt}\,4ta\cos 2\theta = 4a\left(\frac{d}{dt}\,t\cos 2\theta\right)$$

$$= 4a\left(t\,\frac{d}{dt}(\cos 2\theta) + \cos 2\theta\right)$$

$$= 4a\left(t\,\frac{d}{d\theta}(\cos 2\theta)\,\frac{d\theta}{dt} + \cos 2\theta\right)$$

$$= 4a\left(t\,(-\sin(2\theta)\cdot 2)\cdot 2t + \cos 2\theta\right)$$

$$= 4a\left(\cos(2\theta) - 4t^2\sin(2\theta)\right)$$

$$\dot{\theta} = 2t$$

$$\text{so } \ddot{\theta} = \frac{d}{dt}(2t) = 2$$

$$\text{so } \vec{a} = \left(4a\left(\cos(2\theta) - 4t^2\sin(2\theta)\right) - 4t^2 a\,\cancel{\cos(\theta)}\ \sin 2\theta\right)\vec{u}_r$$

$$+ \left(2a\sin(2\theta) + 16t^2 a\cos(2\theta)\right)\vec{u}_\theta$$

$$= \boxed{\left(4a\cos 2\theta - 20t^2 a\sin 2\theta\right)\vec{u}_r + \left(2a\sin(2\theta) + 16t^2 a\cos(2\theta)\right)\vec{u}_\theta}$$