Math 243
Spring 2017
Assignment 3
Selected Solutions:

§11.4:

(6) $\hat{\imath} \times \hat{\jmath} = \hat{k}$

$\hat{\jmath} \times \hat{k} = \hat{\imath}$

$\hat{\imath} \perp \hat{k}$ so $|\hat{\imath} \times \hat{k}| = |\hat{\imath}| \cdot |\hat{k}| = 1 \cdot 1 = \boxed{1}$

& direction is just $\hat{\imath} \times \hat{k} = \boxed{-\hat{\jmath}}$

(16) $\overrightarrow{PQ} = (1, 0, 2)$

$\overrightarrow{PR} = (2, -2, 0)$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & 2 \\ 2 & -2 & 0 \end{vmatrix} = \hat{\imath}\begin{vmatrix} 0 & 2 \\ -2 & 0 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 0 \\ 2 & -2 \end{vmatrix}$$

$$= (4, 4, -2)$$

(a) area of triangle $= \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$

$$= \frac{1}{2}\sqrt{4^2 + 4^2 + (-2)^2}$$

$$= \frac{1}{2}\sqrt{16 + 16 + 4}$$

$$= \frac{1}{2}\sqrt{36}$$

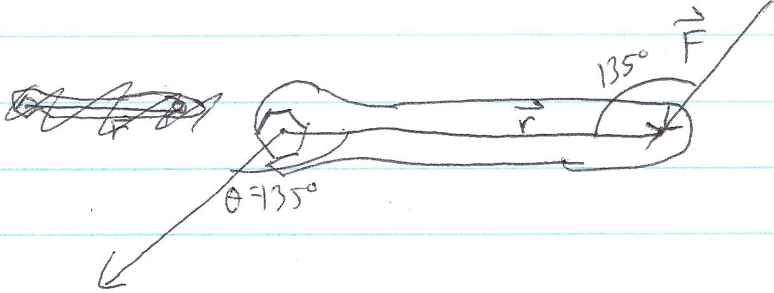
$$= \boxed{3}$$

(b) $\overrightarrow{PQ} \times \overrightarrow{PR}$ is perpendicular to the plane PQR. Its magnitude is 6.

so answer $= \frac{1}{6}(4, 4, -2)$

$$= \boxed{\left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right)}$$

(26)



$|\text{Torque}| = |\vec{r}||\vec{F}| \sin(135°)$

$= 8\text{in} \cdot 30\text{lb} \cdot \frac{\sqrt{2}}{2}$

$= 120\sqrt{2}\ \text{lb} \cdot \text{in}$

$= \boxed{10\sqrt{2}\ \text{ft} \cdot \text{lbs}}$

§11.5:

(36) $\vec{S}=(2,1,-1)$

$x=2t$

$y=1+2t$ so $\vec{v}=(2,2,2)$ & get a point P_0 on the line by

$z=2t$ setting, say, $t=0$. So $P_0=(0,1,0)$

Then $\text{dist}(S, \text{Line}) = \dfrac{|\overrightarrow{P_0S}\times\vec{v}|}{|\vec{v}|}$

$|\vec{v}|=\sqrt{2^2+2^2+2^2}=\sqrt{12}=2\sqrt{3}$

$\overrightarrow{P_0S}=(2,0,-1)$

$\overrightarrow{P_0S}\times\vec{v}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&0&-1\\0&1&0\end{vmatrix}=\hat{i}\begin{vmatrix}0&-1\\1&0\end{vmatrix}\cancel{-\hat{j}\begin{vmatrix}2&-1\\0&0\end{vmatrix}}+\hat{k}\begin{vmatrix}2&0\\0&1\end{vmatrix}$

$=(1,0,2)$

$|\overrightarrow{P_0S}\times\vec{v}|=\sqrt{1^2+2^2}=\sqrt{5}$

so distance $=\boxed{\dfrac{\sqrt{5}}{2\sqrt{3}}}$

Other exercises:

(1)(a) $\vec{r}_0=(0,1,3)$ $\vec{v}=(1,4,-2)-(0,1,3)=(1,3,-5)$

so $\boxed{\vec{r}=(0,1,3)+t\,(1,3,-5)}$ $\boxed{-\infty<t<\infty}$

& $\boxed{\begin{aligned}x&=t\\y&=1+3t\\z&=3-5t\end{aligned}}$

(1)(b) $\vec{r}_0 = (-1,7,0)$ & $\vec{v}(4,-4,4)$ are given

so get $\boxed{\vec{r} = (-1,7,0) + t(4,-4,4)}$

or $\boxed{\begin{aligned} x &= -1 + 4t \\ y &= 7 - 4t \\ z &= 4t \end{aligned}}$ $\boxed{-\infty < t < \infty}$

(c) $\vec{P}_0 = (-3,-2,1)$

Midpoint of the line segment is $\dfrac{(4,6,8)+(8,6,2)}{2} = \dfrac{(12,12,10)}{2}$

$= (6,6,5)$

$\vec{v} = (6,6,5) - (-3,-2,1) = (9,8,4)$

so $\boxed{\vec{r} = (-3,-2,1) + t(9,8,4)}$

$\boxed{\begin{aligned} x &= -3 + 9t \\ y &= -2 + 8t \\ z &= 1 + 4t \end{aligned}}$ $\boxed{-\infty < t < \infty}$

(d) $\dfrac{x-2}{3} = t$ $\boxed{\begin{aligned} x &= 2 + 3t \\ y &= 1 + 5t \\ z &= -1 + 2t \end{aligned}}$ $\boxed{-\infty < t < \infty}$ $\boxed{\vec{r} = (2,1,-1) + t(3,5,2)}$

$\dfrac{y-1}{5} = t$

$\dfrac{z+1}{2} = t$

(3) (a) $\vec{v} = \vec{PQ} = (4,6,5)$ so $\boxed{\vec{r} = (1,0,1) + t(4,6,5) \quad 0 \le t \le 1}$

(b) $\vec{v} = \vec{PQ} = (3,5,-11)$ so $\boxed{\vec{r} = t(3,5,-11) \quad 0 \le t \le 1}$