Math 243
Spring 2017
Asst. 4
Selected solutions

§11.5:

(22) $P_0 = (1,-1,3)$, $\vec{r}_0$, $\vec{n} = (3,1,1)$

$\vec{n} \cdot (x,y,z) = \vec{n} \cdot \vec{r}_0$

$3x + y + z = 3 - 1 + 3$

so $\boxed{3x + y + z = 5}$

(40) $S = (0,0,0)$, $\vec{n} = (3,2,6)$

to find P_0 on the plane, try $y = z = 0$, then $3x = 6$ so $x = 2$

so $P_0 = (2,0,0)$

Then $\text{dist} = \dfrac{|\overrightarrow{P_0S} \cdot \vec{n}|}{|\vec{n}|} = \dfrac{|(-2,0,0) \cdot (3,2,6)|}{\sqrt{3^2+2^2+6^2}}$

$= \dfrac{|-6|}{\sqrt{9+4+36}}$

$= \boxed{\dfrac{6}{7}}$

(42) $\vec{S} = (2,2,3)$ $\vec{n} = (2,1,2)$

P_0: try $y = z = 0$. Then $2x = 4$

so $P_0 = (2,0,0)$

$\text{dist} = \dfrac{|\overrightarrow{P_0S} \cdot \vec{n}|}{|\vec{n}|} = \dfrac{|(0,2,3) \cdot (2,1,2)|}{\sqrt{4+1+4}}$

$= \dfrac{|2+6|}{\sqrt{9}}$

$= \boxed{\dfrac{8}{3}}$

(48) $\vec{n}_1 = (5, 1, -1)$, $\vec{n}_2 = (1, -2, 3)$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{5-2-3}{|\vec{n}_1| \cdot |\vec{n}_2|} = 0$$

so $\boxed{\theta = \frac{\pi}{2}}$

(54) $x = 2$, $y = 3 + 2t$, $z = -2 - 2t$

$$6 \cdot 2 + 3(3+2t) - 4 \cdot (-2-2t) = -12$$
$$12 + 9 + 6t + 8 + 8t = -12$$
$$14t + 29 = -12$$
$$14t = -41$$
$$t = \frac{-41}{14}$$

so $x = 2$, $y = 3 + 2 \cdot \left(\frac{-41}{14}\right) = 3 - \frac{41}{7}$, $z = -2 - 2 \cdot \left(\frac{-41}{14}\right) = -2 + \frac{41}{7}$

so point of intersection is $\boxed{\left(2, \frac{-20}{7}, \frac{27}{7}\right)}$

(58) $\vec{n}_1 = (3, -1, -2)$, $\vec{n}_2 = (2, 1, -2)$

$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -2 \\ 2 & 1 & -2 \end{vmatrix} = \hat{i}\begin{vmatrix} -1 & -2 \\ 1 & -2 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & -2 \\ 2 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix}$$
$$= (2+2, -(-6+4), 3+2)$$
$$= (4, 2, 5)$$

P_0: Solve $3x - 6y - 2z = 3$, $2x + y - 2z = 2$

try $z = 0$, then $3x - 6y = 3$

so $3x = 3 + 6y$

so $x = 1 + 2y$

so $2 \cdot (1 + 2y) + y = 2$

so $2 + 4y + y = 2$

so $y = 0$ so $x = 1$

so $P_0 = (1, 0, 0)$

so line is $\boxed{\vec{r} = (1,0,0) + t(4,2,5)}$

Other questions:

(1) $\vec{u} = \overrightarrow{PQ} = (1, -2, -3)$

$\vec{v} = \overrightarrow{PR} = (-4, -3, -10)$

$\vec{r}_0 = \overrightarrow{OP} = (1, 1, 5)$

Then $\boxed{\vec{r} = (1, 1, 5) + s(1, -2, -3) + t(-4, -3, -10)}$

(3) $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{r}_0$

$4x - y + 2z = (4, -1, 2) \cdot (1, 3, 7)$

$= 4 - 3 + 14$

so $\boxed{4x - y + 2z = 15}$