# Math 243
## Spring 2017
## Asst 5
## Solutions

§3.9

(22) $\left.\frac{dy}{dx}\right|_{t=2\pi/3} = \left.\frac{dy/dt}{dx/dt}\right|_{t=2\pi/3}$

$\frac{dy}{dt} = -\sqrt{3}\sin(t)$

$\frac{dx}{dt} = -\sin(t)$

$= \sqrt{3}\Big|_{t=2\pi/3}$

$= \sqrt{3}$

$\vec{r}(2\pi/3) = \left(-\frac{1}{2}, -\sqrt{3}/2\right)$

so tangent is $\left(y + \sqrt{3}/2\right) = \sqrt{3}\left(x + \frac{1}{2}\right)$

$y = \sqrt{3}x - \sqrt{3}/2 + \sqrt{3}/2$

$\boxed{y = \sqrt{3}x}$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(y')}{dx/dt}$

$\frac{d}{dt}(y') = \frac{d}{dt}(\sqrt{3}) = 0$

so $\left.\frac{d^2y}{dx^2}\right|_{t=2\pi/3} = \boxed{0}$

(26) $\frac{dy}{dt} = \sin(t)$ $\quad \frac{dx}{dt} = 1 - \cos t$

$\left.\frac{dy}{dx}\right|_{t=\pi/3} = \left.\frac{\sin(t)}{1-\cos t}\right|_{t=\pi/3}$

$= \frac{\sqrt{3}/2}{1-1/2} = \sqrt{3}$

$\vec{r}(\pi/3) = \left(\frac{\sqrt{3}}{2}, 1-\frac{1}{2}\right)$

$= \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

tangent is $\left(y - \frac{1}{2}\right) = \sqrt{3}\left(x - \frac{\sqrt{3}}{2}\right)$

$y = \frac{1}{2} + \sqrt{3}x - 3/2$

$\boxed{y = \sqrt{3}x - \frac{1}{2}}$

$\frac{d}{dt}y' = \frac{(1-\cos(t))\cos t - \sin t(\sin t)}{(1-\cos t)^2} = \frac{\cos(t)}{(1-\cos t)^2}$

so $\left.\frac{d^2y}{dx^2}\right|_{t=\pi/3} = \left.\frac{\frac{\cos(t)}{(1-\cos t)^2}}{(1-\cos t)}\right|_{t=\pi/3} = \frac{1/2}{(1-1/2)^3} = \left(\frac{1}{2}\right)^{-2} = \boxed{4}$

§11.6

(2) (i)
(4) (g)
(6) (e)
(8) (j)
(10) (f)
(12) (c)

Other exercises:

(2) $\vec{r}(t) = (t, t^2 - 2t + 1)$ $-2 \le t \le 2$

11.6 #1-12 even

2. $z^2+4y^2-4x^2=4$

- ellipse $\frac{z^2}{4}+y^2=1$ in yz-plane
- hyperbola $\frac{z^2}{4}-x^2=1$ in xz-plane
  $y^2-x^2=1$ in xy-plane

note: origin not in graph

y & z can't both be zero
⇒ x-axis not in graph

(i) hyperboloid

4. $y^2+z^2=x^2$

- (0,0,0) only point in yz-plane
- lines $z=x$, $z=-x$ in xz-plane
- lines $y=x$, $y=-x$ in xy-plane
- circle $y^2+z^2=k^2$ in $x=k$ plane

(g) cone

6. $x=-y^2-z^2$

- parabola $x=-y^2$ in xy-plane
  $x=-z^2$ in xz-plane
- pt (0,0,0) only pt in yz-plane

(e) paraboloid

8. $z^2+x^2-y^2=1$

- circle $z^2+x^2=1$ in xz-plane
- hyperbola $z^2-y^2=1$ in yz-plane
  $x^2-y^2=1$ in xy-plane

note: x and z can't both be zero
⇒ y-axis not in graph (j) hyperboloid

10. $z=-4x^2-y^2$

- parabola $z=-4x^2$ in xz-plane
  $z=-y^2$ in yz-plane
- (0,0,0) only pt in xy-plane (f) paraboloid

12. $9x^2+4y^2+2z^2=36$

- ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ in xy-plane (c) ellipsoid
  $\frac{x^2}{4}+\frac{z^2}{18}=1$ in xz-plane
  $\frac{y^2}{9}+\frac{z^2}{18}=1$ in yz-plane
  ↑ longest in z-direction