Math 243

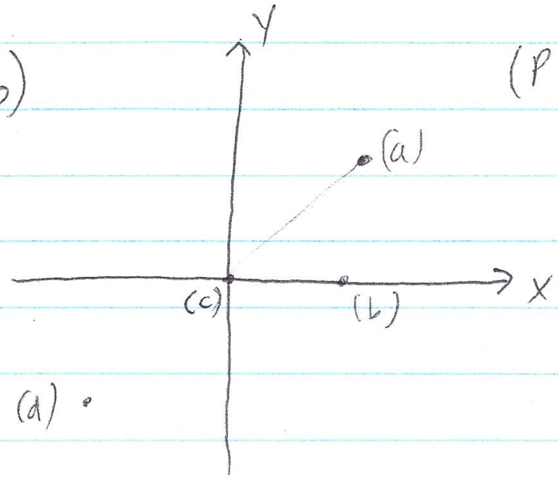
Spring 2017

Asst 7

Selected solutions

§10.1:

(6)



(Plot is not required, but helpful)

(a) $x = r\cos\theta = \sqrt{2}\cos(\pi/4) = \sqrt{2}\cdot\frac{\sqrt{2}}{2} = \boxed{1}$

$y = r\sin\theta = \sqrt{2}\sin(\pi/4) = \sqrt{2}\cdot\frac{\sqrt{2}}{2} = \boxed{1}$

(b) $x = r\cos\theta = 1\cdot\cos(0) = \boxed{1}$

$y = r\sin\theta = 1\cdot\sin(0) = \boxed{0}$

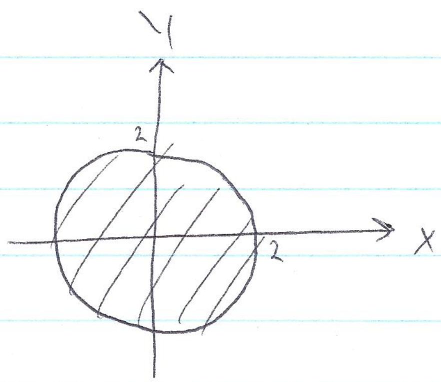
(c) $x = 0\cdot\cos(\pi/2) = \boxed{0}$

$y = 0\cdot\sin(\pi/2) = \boxed{0}$

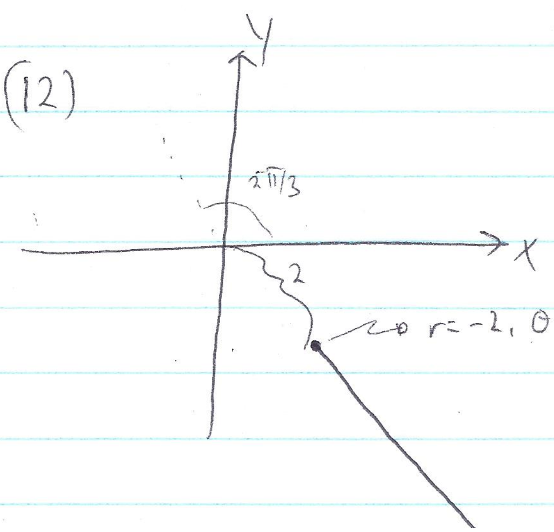
(d) $x = r\cos\theta = -\sqrt{2}\cdot\frac{\sqrt{2}}{2} = \boxed{-1}$

$y = r\sin\theta = -\sqrt{2}\cdot\frac{\sqrt{2}}{2} = \boxed{-1}$

(8)



(12)



← r = −2, θ = 2π/3 so $x = -2\cos 2\pi/3 = -1$

$y = -2\sin(2\pi/3) = -\sqrt{3}$

§10.2:

(18) $\dfrac{dy}{dx} = \dfrac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$

here $f(\theta) = -1+\sin\theta$

$f'(\theta) = \cos\theta$

$$\frac{dy}{dx} = \frac{\cos\theta\sin\theta + (-1+\sin\theta)\cos(\theta)}{\cos\theta\cos\theta - (-1+\sin\theta)\sin(\theta)} = \frac{2\sin\theta\cos\theta - \cos\theta}{\cos^2\theta - \sin^2\theta\,\cancel{\cos\theta} + \cancel{\cos\theta}\sin\theta}$$

at $\theta=0$: $\left.\dfrac{dy}{dx}\right|_{\theta=0} = \dfrac{0-1}{1-0+1} = \boxed{-\dfrac{1}{2}}$

at $\theta=\pi$: $\left.\dfrac{dy}{dx}\right|_{\theta=\pi} = \dfrac{0+1}{1-0\pm0} = \boxed{1}$

§10.3:

(18) $L = \displaystyle\int_0^\pi \sqrt{r(\theta)^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Step 1: find $\dfrac{dr}{d\theta}$

here $r(\theta) = \dfrac{e^\theta}{\sqrt{2}}$ so $\dfrac{dr}{d\theta} = \dfrac{e^\theta}{\sqrt{2}}$

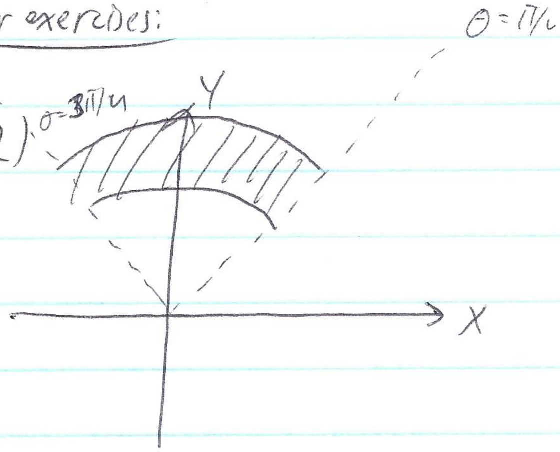
Step 2: Plug into formula

$$L = \int_0^\pi \sqrt{\left(\frac{e^\theta}{\sqrt{2}}\right)^2 + \left(\frac{e^\theta}{\sqrt{2}}\right)^2}\, d\theta = \int_0^\pi \sqrt{\frac{e^{2\theta}}{2} + \frac{e^{2\theta}}{2}}\, d\theta = \int_0^\pi \sqrt{e^{2\theta}}\, d\theta = \int_0^\pi e^\theta\, d\theta$$

Step 3: Evaluate the integral

$$L = \int_0^\pi e^\theta\, d\theta = e^\theta\Big|_0^\pi = e^\pi - e^0 = \boxed{e^\pi - 1}$$

Other exercises:

(2) 

(4) $A=\int_0^{\pi/4}\frac{1}{2}\left(f(\theta)^2-g(\theta)^2\right)d\theta$ here $f(\theta)=2\cos\theta$

$g(\theta)=\cos\theta$

so $A=\int_0^{\pi/4}\frac{1}{2}\left(4\cos^2\theta-\cos^2\theta\right)d\theta$

$=\int_0^{\pi/4}\frac{1}{2}\left(3\cos^2\theta\right)d\theta$

$=\frac{3}{2}\int_0^{\pi/4}\cos^2\theta\, d\theta$

$=\frac{3}{2}\int_0^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)d\theta$

$=\frac{3}{2}\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\Big|_0^{\pi/2}\right)$

$=\frac{3}{2}\left(\frac{\pi}{4}+\frac{\sin\pi}{4}-\left(\frac{0}{2}+\frac{\sin 0}{4}\right)\right)=\frac{3}{2}\cdot\frac{\pi}{4}=\boxed{\frac{3\pi}{8}}$