Math 243
Spring 2017
Asst 8
Selected solutions

(10) $\vec{v}(t) = \frac{d\vec{r}}{dt} = \left(t, \frac{2t}{\sqrt{2}}, \frac{\cancel{3}t^2}{\cancel{3}}\right)$

so $\boxed{\vec{v}(t) = \left(t, \sqrt{2}t, t^2\right)}$

$\boxed{\vec{a}(t) = \frac{d\vec{v}}{dt} = \left(1, \sqrt{2}, 2t\right)}$

at $t=1$, speed $= |\vec{v}(1)| = |(1, \sqrt{2}, 1)| = \sqrt{1+2+1} = \sqrt{4} = \boxed{2}$

so at $t=1$, direction of motion is $\hat{v}(1) = \frac{\vec{v}(1)}{2}$

$= \boxed{\left(\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2}\right)}$

so $\boxed{\vec{v}(1) = 2 \cdot \left(\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2}\right)}$

(16) $\vec{v}(t) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - 32t\right)$

$\vec{a}(t) = (0, -32)$

$\vec{v}(0) \cdot \vec{a}(0) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot (0, -32)$

$= -16\sqrt{2}$

$|\vec{v}(0)| = \sqrt{\frac{2}{4} + \frac{2}{4}} = 1$

$|\vec{a}(0)| = 32$

so $\cos\theta = \frac{-16\sqrt{2}}{1 \cdot 32} = -\frac{\sqrt{2}}{2}$

so $\boxed{\theta = \frac{3\pi}{4}}$

(20) $\vec{r}'(t) = (2\cos(t), -2\sin(t), 5)$

$\vec{r}'(4\pi) = (2, 0, 5)$

$\vec{r}(4\pi) = (0, 2, 20\pi)$

so tangent line is $\boxed{(0, 2, 20\pi) + t(2, 0, 5)}$

Other exercises:

(1) (a) $\int (t^2 - 2, 1/t, -2)\,dt = \boxed{\left(\frac{t^3}{3} - 2t, \ln|t|, -2t\right) + \vec{C}}$

(b) $\int_0^1 (2\sqrt{t}, 1-2t, e^t)\,dt = \left(\frac{4t^{3/2}}{3}\Big|_0^1, t - t^2\Big|_0^1, e^t\Big|_0^1\right)$

$= \boxed{\left(\frac{4}{3}, 0, e\right)}$

(2) $\vec{r}(t) = \int \vec{v}(t)\,dt$

$= \int (e^t - 1, -t, \sqrt{t})\,dt$

$= \left(e^t - t, -\frac{t^2}{2}, \frac{2t^{3/2}}{3}\right) + \vec{C}$

$\underset{\vec{0}}{\underset{\parallel}{\vec{r}(0)}} = (1 - 0, 0, 0) + \vec{C}$

so $\vec{C} = (-1, 0, 0)$

so $\boxed{\vec{r}(t) = (-1, 0, 0) + \left(e^t - t, -\frac{t^2}{2}, \frac{2t^{3/2}}{3}\right)}$

(3) $\vec{r}'(t) = (1, 2t, 3t^2)$

so $L = \int_1^3 \sqrt{1^2 + (2t)^2 + (3t^2)^2}\,dt$

$= \boxed{\int_1^3 \sqrt{1 + 4t^2 + 9t^4}\,dt}$