Math 243
Spring 2017
Asst 9
Selected Solutions

§12.3

(12) $\vec{r}(t) = (6\sin(2t), 6\cos(2t), 5t) \quad 0 \le t \le \pi$

$\vec{v}(t) = (6\cos(2t)\cdot 2, -6\sin(2t)\cdot 2, 5)$

$|\vec{v}(t)| = \sqrt{144\cos^2(2t) + 144\sin^2(2t) + 25}$

$= \sqrt{169}$

$= 13$

so $\vec{T}(t) = \frac{1}{13}\vec{v}(t) = \boxed{\left(\frac{12}{13}\cos(2t), -\frac{12}{13}\sin(2t), \frac{5}{13}\right)}$

Arc length:

Step 1: $\vec{r}\,'(t) = (12\cos(2t), -12\sin(2t), 5)$ from above

Step 2: Formula: $L = \int_0^{\pi} |v(t)|\,dt$

$= \int_0^{\pi} 13\,dt$ from above

Step 3: Evaluate $= \boxed{13\pi}$

(14) Step 1: $v(t) = (2, 3, -6)$

Step 2: $s(t) = \int_0^t |\vec{v}(\tau)|\,d\tau$

$= \int_0^t \sqrt{2^2+3^2+(-6)^2}\,d\tau$

Step 3: Evaluate $= \int_0^t \sqrt{49}\,d\tau$

$= \int_0^t 7\,d\tau$

$= 7t$

so $\boxed{s(t) = 7t}$

so arc length from $t=-1$ to $t=0$ is $s(0) - s(-1) = 0 - -7 = \boxed{7}$

§12.4

(10) $\vec{T}$: $\vec{v}(t) = (-\sin(t) + t\cos t + \sin(t),\ \cos(t) + t\sin(t) - \cos(t),\ 0)$

$= (t\cos(t), t\sin(t), 0)$

so $|\vec{v}(t)| = \sqrt{t^2\cos^2(t) + t^2\sin^2(t)}$

$= \sqrt{t^2}$

$= t$ $\quad (t \geq 0)$

so $\boxed{\vec{T}(t) = (\cos(t), \sin(t), 0)}$

$\vec{N}$: $\frac{d\vec{T}}{dt} = (-\sin(t), \cos(t), 0)$ so $\left|\frac{d\vec{T}}{dt}\right| = 1$

so $\vec{N}(t) = \frac{\frac{d\vec{T}}{dt}}{|d\vec{T}/dt|} = \boxed{(-\sin(t), \cos(t), 0)}$

κ: $\kappa = \frac{1}{|v(t)|}\left|\frac{d\vec{T}}{dt}\right|$

$= \frac{|(-\sin(t), \cos(t), 0)|}{t}$ $\quad (t > 0)$

$= \boxed{\frac{1}{t}}$

Other:

(1) (a) Step 1: $\vec{r}\,'(t) = (-1, 2, 7)$

Step 2: $s(t) = \int_0^t \sqrt{|\vec{r}\,'(\tau)|}\, d\tau$

$= \int_0^t \sqrt{1^2 + 2^2 + 7^2}\, d\tau$

Step 3: Evaluate: $s(t) = \int_0^t \sqrt{54}\, d\tau = \sqrt{54}\, t$

so $s(t) = \sqrt{54}\, t$ so $t(s) = \frac{s}{\sqrt{54}}$

so $\boxed{\vec{r}(t(s)) = \left(5 - \frac{s}{\sqrt{54}}, -1 + \frac{2s}{\sqrt{54}}, \frac{7s}{\sqrt{54}}\right)}$

(1) (b) Step 1: $\vec{r}\,'(t) = (-10\sin(t), 10\cos(t), 0)$

Step 2: $s(t) = \int_0^t \sqrt{100\sin^2(\tau) + 100\cos^2(\tau)}\, d\tau$

Step 3: Eval: $= \int_0^t \sqrt{100}\, d\tau$

$= 10t$

So $s(t) = 10t$ so $t(s) = \frac{s}{10}$

So $\boxed{\vec{r}(t(s)) = \left(10\cos\left(\frac{s}{10}\right), 10\sin\left(\frac{s}{10}\right), 5\right)}$