Assignment 12 – All 2 parts – Math 411

Due in the class: Friday, Apr. 24, 2015

(1) Find the determinants of the following matrices.

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{pmatrix}.$$

(2) The Vandermonde Determinant: Find the determinant of

$$\begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{pmatrix}.$$

(3) (a) Suppose A is an invertible matrix. Show that $\det(A^{-1}) = \det(A)^{-1}$.

(b) Show that the determinant of an orthogonal matrix is ± 1.

(c) Show that the determinant of a unitary matrix is a complex number of absolute value 1.

(d) Show that the determinant of a Hermitian matrix is real.

(4) Use Cramer's rule to find the solution the linear system $Ax = b$ where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

(5) Let A be the $n \times n$ matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & c_0 \\ 1 & 0 & 0 & \cdots & 0 & c_1 \\ 0 & 1 & 0 & \cdots & 0 & c_2 \\ 0 & 0 & 1 & \cdots & 0 & c_3 \\ & \vdots & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 & c_{n-1} \end{pmatrix}$$

(i.e. there are 1's right below the diagonal, and the last column has entries $c_0, c_1, \ldots, c_{n-1}$).

(a) Show that $\det(A) = (-1)^{n+1} c_0$.

(b) Let I denote the $n \times n$ identity matrix and let x denote a variable. Show that $\det(xI - A) = x^n - c_{n-1}x^{n-1} - c_{n-2}x^{n-2} - \cdots - c_1 x - c_0$. (Hint: one way to do this is Laplace expansion along the first row together with induction.)