Assignment 1 – Part 1 – Math 411

(1) Consider the following three vectors in $\mathbf{C}^2$

$$v_1 = \begin{pmatrix} 2-i \\ 5 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 3+i \end{pmatrix}, \quad w = \begin{pmatrix} 3-3i \\ 8+2i \end{pmatrix}.$$

Determine the solutions, if any, to the equation

$$\alpha_1 v_1 + \alpha_2 v_2 = w$$

with $\alpha_i \in \mathbf{C}$.

(2) Recall the following addition and multiplication tables given in class for the field $\mathbf{F}_2$ with two elements

$+$	0	1
0	0	1
1	1	0

$\cdot$	0	1
0	0	0
1	0	1

Taking for granted that this gives a field, consider the following three vectors in $\mathbf{F}_2^3$

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

For each of

$$w = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

determine the solutions, if any, to the equation

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = w$$

with $\alpha_i \in \mathbf{F}_2$.

(3) Let F be a field. Show that if there is a non-zero element $\alpha \in F$ such that $\alpha + \alpha = 0$, then $\beta + \beta = 0$ for all $\beta \in F$. (Hint: first show that $1 + 1 = 0$)