Assignment 2 – Part 1 – Math 411

(1) A matrix A is said to be in reduced row echelon form (RREF) if it is in row echelon form and it satisfies the following two properties:

(RREF1) Every entry lying above a leading entry is 0.

(RREF2) Every leading entry is 1.

This exercise will walk you through the proof that every $m \times n$ matrix A is row-equivalent to an $m \times n$ matrix that is in reduced row echelon form.

(a) Suppose A is in row echelon form. Explain why doing any elementary row operation of type (III) (i.e. $r_i \rightsquigarrow \alpha r_i$, $\alpha \neq 0$) leaves A in row echelon form. Explain why doing an elementary row operation of type (II) sending $r_i \rightsquigarrow r_i + \alpha r_j$ leaves A in row echelon form if $i < j$.

(b) Use induction on the number of rows of A to show that A is row-equivalent to a matrix in RREF. (Hint: first, let A' be a matrix in REF that is row-equivalent to A. Then, proceed similarly to how we did in class for row echelon form and deal with the first row and use induction for the remaining rows. Part (a) of this question will be relevant, so don't forget to explain how it matters.)

(2) Suppose A is an $m \times n$ matrix in RREF and B is an $m \times k$ matrix in RREF. Show that the $m \times (n + k)$ matrix obtained by placing B to the right of A, in notation $(A\ B)$, is in RREF.

(3) Let F be any field. Write down all RREF 2×2 matrices over F.

(4) Let $F = \mathbf{F}_2$ be the field with two elements. Write down all REF 2×3 matrices over F. Circle the ones that are in RREF.