Assignment 4 – Part 1 – Math 411

(1) Let $F = \mathbf{F}_2$ and let V be a two-dimensional vector space over F.

(a) How many vectors are there in V?

(b) How many subspaces of V are there?

(c) How many bases of V are there?

(d) How many bases of a three-dimensional vector space over F are there?

(e) How many bases of a two-dimensional vector space over $\mathbf{F}_3$ are there?

(2) Let F be a field and let $m, n \in \mathbf{Z}_{\geq 1}$. Let $M_{m,n}(F)$ denote the set of $m \times n$ matrices over F (i.e. with entries in F). For two matrices $A = (a_{ij})$ and $B = (b_{ij})$ in $M_{m,n}(F)$, let $A + B = (a_{ij} + b_{ij})$ be the usual matrix addition and $\alpha A = (\alpha a_{ij})$ be the usual scalar multiplication with $\alpha \in F$. This makes $M_{m,n}(F)$ into a vector space over F (no need to show this, though you can think about why).

(a) Find a basis for $M_{m,n}(F)$.

(b) What is the dimension of $M_{m,n}(F)$?