

## Assignment 5 – Part 1 – Math 411

- (1) (a) Let  $F = \mathbf{R}$ . Show that the set of solutions to a system of linear equations over  $F$  has either 0, 1, or infinitely many solutions. Write down a system that has each number of solutions.  
(b) Let  $F = \mathbf{F}_2$ . How many solutions can a system of linear equations in two variables over  $F$  have? Write down a system that has each number of solutions.
- (2) Can a homogeneous system of two linear equations in four variables have a one-dimensional solution space?
- (3) (a) Suppose  $\mathcal{S}$  denotes a homogeneous system of equations in 10 variables whose solution space is 2-dimensional and  $\mathcal{S}'$  denotes a homogeneous system of equations in 10 variables whose solution space is 5-dimensional. What are the possible dimensions of the solution space of the combined systems? (I.e. of the system consisting of all the equations of both of the original systems.)  
(b) Suppose that  $\mathcal{S}$  and  $\mathcal{S}'$  are systems in 6 variables instead. Show that they share a non-trivial solution.