Assignment 5 – All 2 parts – Math 411

Due in the class: Friday, Feb. 20, 2015

(1) (a) Let $F = \mathbf{R}$. Show that the set of solutions to a system of linear equations over F has either $0, 1$, or infinitely many solutions. Write down a system that has each number of solutions.

(b) Let $F = \mathbf{F}_2$. How many solutions can a system of linear equations in two variables over F have? Write down a system that has each number of solutions.

(2) Can a homogeneous system of two linear equations in four variables have a one-dimensional solution space?

(3) (a) Suppose $\mathcal{S}$ denotes a homogeneous system of equations in 10 variables whose solution space is 2-dimensional and $\mathcal{S}'$ denotes a homogeneous system of equations in 10 variables whose solution space is 5-dimensional. What are the possible dimensions of the solution space of the combined systems? (I.e. of the system consisting of all the equations of both of the original systems.)

(b) Suppose that $\mathcal{S}$ and $\mathcal{S}'$ are systems in 6 variables instead. Show that they share a non-trivial solution.

(4) Let $F = \mathbf{R}$. Find a basis for the solution space of the homogeneous system of linear equations whose coefficient matrix is

$$\begin{pmatrix} 0 & 0 & 0 & 5 & 0 \\ 0 & 2 & 4 & 0 & 6 \\ 0 & -2 & -4 & 3 & -8 \end{pmatrix}.$$

(5) Let $F = \mathbf{R}$. You purchased a 3×4 matrix on ebay, but when it was delivered you noticed one of the entries was missing:

$$\begin{pmatrix} 1 & 2 & -1 & 4 \\ 1 & 4 & 2 & 6 \\ 1 & 4 & ? & -1 \end{pmatrix}.$$

Luckily, you remember that this is meant to be the augmented matrix of a linear system with no solutions. What is the missing entry?

(6) Let $F = \mathbf{R}$. Write down a system of linear equations whose solution space is the span of the vectors

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 5 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} -4 \\ 0 \\ 0 \\ 6 \\ 0 \\ 2 \end{pmatrix}.$$

(Feel free to write down the augmented matrix corresponding to the system, rather than the system itself.)

(7) Let $F = \mathbf{F}_2$. Write down a system of two linear equations over F in three unknowns that has no solution.

(8) (a) Let A be an $m \times n$ matrix over a field F. Show that the set of vectors $b \in F^m$ such that linear system $(A \;\; b)$ has a solution is subspace of F^m.

(b) Let $F = \mathbf{R}$ and let

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 6 & 1 \\ 1 & 2 & 1 \\ -1 & -2 & 3 \end{pmatrix}.$$

Find a basis for the space of all $b \in \mathbf{R}^4$ such that $(A \;\; b)$ has a solution.