Assignment 8 – Parts 1 & 2 – Math 411

(1) Let V be a vector space over F, let $\alpha \in F$, and let $T_\alpha : V \to V$ be the linear transformation $v \mapsto \alpha v$. Prove the following properties we discussed in class.

(a) $T_0 = 0$

(b) $T_1 = 1$

(c) $T_{\alpha_1+\alpha_2} = T_{\alpha_1} + T_{\alpha_2}$

(d) $T_{\alpha_1\alpha_2} = T_{\alpha_1} T_{\alpha_2}$

(Remark: these show that F can be thought of as a "subalgebra" of $\mathrm{End}(V)$.)

(2) Let $T : V \to W$ be a linear transformation. Prove the following statements (without assuming V and W are finite-dimensional).

(a) If $\dim(V) < \dim(W)$, then T cannot be surjective.

(b) If $\dim(V) > \dim(W)$, then T cannot be injective.