Assignment 11 – Part 1 – Math 412

Textbook exercises:[1]

Section 5.3: 8 (WI), 11

Other exercises:

(1) Recall that in class we considered the field $\mathbf{F}_4 = (\mathbf{Z}/2\mathbf{Z})[x]/(x^2 + x + 1)$. Let $f_4(x) = x^4 - x \in \mathbf{F}_4[x]$. Show that every element of $\mathbf{F}_4$ is a root of $f_4(x)$. (Remark: and so, since a polynomial of degree n has at most n roots in a field, $\mathbf{F}_4$ is exactly the set of all roots of $f_4(x)$ in itself.)

(2) Write out the multiplication table for the ring $(\mathbf{Z}/2\mathbf{Z})[x]/(x^3 + x + 1)$. Is this ring a field?

(3) Write out the multiplication table for the ring $(\mathbf{Z}/2\mathbf{Z})[x]/(x^3 + x^2 + x + 1)$. Is this ring a field?

[1]From Hungerford's *Abstract algebra, An introduction, Third edition*