Assignment 13 – Part 1 – Math 412

Textbook exercises:[1]

Section 6.1: 8, 28

Other exercises:

(1) (WI) Let $\varphi : R \to S$ be a ring homomorphism and let J be an ideal in S. Show that $\varphi^{-1}(J) = \{r \in R : \varphi(r) \in J\}$ is an ideal of R.

(2) Let $R = \mathbf{Z}/6\mathbf{Z}$.

  (a) What are the ideals of R?

  (b) For each ideal I of R, determine R/I.

(3) Let R be a ring and let I and J be ideals of R. Let IJ denote the smallest ideal of R containing $\{ab : a \in I, b \in J\}$. (This latter set is not always itself an ideal: for instance, if $R = \mathbf{Z}[x], I = (2, x)$, and $J = (3, x)$, then IJ contains $3x - 2x = x$, but $x \notin \{ab : a \in I, b \in J\}$).

  (a) Show that $IJ = \{a_1b_1 + a_2b_2 + \cdots + a_nb_n : n \in \mathbf{Z}_{\geq 1}, a_i \in I, b_i \in J\}$.

  (b) Show that if R is commutative and I and J are principal, then

$$IJ = \{ab : a \in I, b \in J\}.$$

[1]From Hungerford's *Abstract algebra, An introduction, Third edition*

TURN OVER