Assignment 1 – All 2 parts – Math 412

Due: Thursday, Sept. 1, 2016, at the beginning of class

Textbook exercises:[1]

Section 1.1: 6 (WI)

Section 1.2: 6, 8, 11, 16 (WI), 25

Other exercises:

(1) Sums of three squares. In class, we showed that if $n \equiv 3 \pmod 4$, then n cannot be written as a sum of two squares. This question gives a similar result for sums of three squares.

   (a) Show that the square of any integer is either $0, 1$, or 4 modulo 8.

   (b) Conclude that if $n \equiv 7 \pmod 8$, then n cannot be written as a sum of three squares.

   (c) Give an example of a number $n \equiv 3 \pmod 4$ that *is* a sum of three squares.

(2) (WI) Let a and b be positive integers, and let $g = (a, b)$. Suppose $u_0, v_0 \in \mathbf{Z}$ are such that $g = au_0 + bv_0$.

   (a) Let $k \in \mathbf{Z}$ and let
   $$u = u_0 + \frac{bk}{g} \quad \text{and} \quad v = v_0 - \frac{ak}{g}.$$
   Show that $au + bv = g$.

   (b) Conversely, show that if $g = au + bv$ with $u, v \in \mathbf{Z}$, then there is an integer k such that
   $$u = u_0 + \frac{bk}{g} \quad \text{and} \quad v = v_0 - \frac{ak}{g}.$$

[1] From Hungerford's *Abstract algebra, An introduction, Third edition*