Assignment 7 – All 2 parts – Math 412

Due in class: Thursday, Oct. 17, 2019

Textbook exercises:[1]

Section 4.4: 8, 13, 16(WI), 18

Section 4.5: 3, 5, 6

Other exercises:

(1) List all monic irreducible polynomials in $(\mathbf{Z}/3\mathbf{Z})[x]$ of degree ≤ 3.

(2) (WI) Let $n \in \mathbf{Z}_{\geq 1}$ and define a function $\mathbf{Z}[x] \to (\mathbf{Z}/n\mathbf{Z})[x]$ written as $f(x) \mapsto \overline{f}(x)$ obtained by reducing the coefficients of $f(x)$ modulo n. So, for instance, if $n = 3$ and $f(x) = 5x^3 - 3x^2 + 10x + 1$, then

$$\overline{f}(x) = [2]x^3 + x + 1.$$

We call this the reduction mod n map on $\mathbf{Z}[x]$.

(a) Show that the reduction mod n map is a ring homomorphism.

(b) Show that $\deg(\overline{f}) \leq \deg(f)$ with equality if and only if n does not divide the leading coefficient of f (or $f = 0$).

(c) Recall that when p is prime, $\mathbf{Z}/p\mathbf{Z}$ is a field.

[1] From Hungerford's *Abstract algebra, An introduction, Third edition*