Assignment 8 – Part 1 – Math 412

Textbook exercises:[1]

Section 6.1: 9, 10, 20(WI)

Section 6.2: 10, 18

Other exercises:

(1) (WI) Let p be a prime and let $I = (p, x)$ be the ideal generated by p and x in $R = \mathbf{Z}[x]$. Show that $R/I \cong \mathbf{Z}/p\mathbf{Z}$.

(2) Let p be a prime number and let $R = \mathbf{Z}_{(p)}$ be the ring from Assignment 4, Question 1 (and Assignment 6, Question 3(c)). Let $I = p\mathbf{Z}_{(p)}$ be the principal ideal generated by p in R. Show that $R/I \cong \mathbf{Z}/p\mathbf{Z}$. (Hint: you can start by showing that for $a/b \in \mathbf{Z}_{(p)}$, $\varphi(a/b) := [a] \cdot [b]^{-1} \in \mathbf{Z}/p\mathbf{Z}$ make sense and gives a ring homomorphism.)

[1] From Hungerford's *Abstract algebra, An introduction, Third edition*