Assignment 9 – All 2 parts – Math 412

Due in class: Thursday, Nov. 14, 2019

Textbook exercises:[1]

Section 7.1: 7(a), 16, 29

Section 7.2: 7

Other exercises:

(1) From the Chinese Remainder Theorem, $\mathbf{Z}/12\mathbf{Z} \cong \mathbf{Z}/4\mathbf{Z} \times \mathbf{Z}/3\mathbf{Z}$. Write out this bijection explicitly, i.e. for each $a \in \mathbf{Z}/12\mathbf{Z}$, write out the element it corresponds to in $\mathbf{Z}/4\mathbf{Z} \times \mathbf{Z}/3\mathbf{Z}$.

(2) We saw in class that if R is a commutative ring and I, J are ideals in R, then $IJ \subseteq I \cap J$. Show that $(I \cap J)^2 \subseteq IJ$ (where $(I \cap J)^2$ just means the product ideal $(I \cap J)(I \cap J)$).

(3) (WI) Show that the identity element in a group G is unique, i.e if $e, e' \in G$ are such that for all $g \in G, g \cdot e = e \cdot g = g = g \cdot e' = e' \cdot g$, then $e = e'$. Show that for every $g \in G$, there is a unique element $h \in G$ such that $g \cdot h = e = h \cdot g$.

(4) (WI) Let G be a group and fix an element $h \in G$.

(a) Define a function $m_h : G \to G$ by $m_h(g) = hg$ (this is called the *left multiplication by h function*). Show that m_h is a bijection.

(b) Define a function $c_h : G \to G$ by $m_h(g) = hgh^{-1}$ (this is called the *left conjugation by h function*). Show that c_h is a bijection.

[1] From Hungerford's *Abstract algebra, An introduction, Third edition*