Assignment 11 – Part 1 – Math 612

Throughout, assume all rings are commutative.

(1) Let R be a commutative ring. Recall that the polynomial ring $R[x,y]$ satisfies a universal property stemming from the fact that it is the free commutative R-algebra on two elements, i.e. for every R-algebra A, there is a natural bijection

$$\mathrm{Hom}_{R\text{-alg}}(R[x,y],A) \cong \mathrm{Hom}_{\mathbf{Set}}(\{x,y\},A).$$

In class, we stated that $R[x] \otimes_R R[y] \cong R[x,y]$. Prove this by showing that $R[x] \otimes_R R[y]$ satisfies the same universal property as $R[x,y]$.

(2) Determine the following tensor products.

(a) $\mathbf{Q}(i) \otimes_{\mathbf{Q}} \mathbf{R}$

(b) $\mathbf{Q}(i) \otimes_{\mathbf{Q}} \mathbf{C}$

(c) $\mathbf{Q}(i) \otimes_{\mathbf{Q}} \mathbf{Q}(i)$

(d) $\mathbf{Q}(i) \otimes_{\mathbf{Q}} \mathbf{Q}(\sqrt{2})$

(e) $\mathbf{Z}[i] \otimes_{\mathbf{Z}} \mathbf{F}_2$

(f) $\mathbf{Z}[i] \otimes_{\mathbf{Z}} \mathbf{F}_3$

(g) $\mathbf{Z}[i] \otimes_{\mathbf{Z}} \mathbf{F}_5$