Assignment 10 – All 1 part – Math 612

Due in class: Thursday, Apr. 18, 2019

(1) Recall from class that a *transitive subgroup* of S_n is a subgroup of S_n that has only one orbit when acting on $\{1, 2, \ldots, n\}$.

   (a) Let $H = \{1, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\} \leq S_4$. Show that H is a transitive subgroup of S_4 isomorphic to $V_4 = C_2 \times C_2$.

   (b) Consider S_3 acting on itself by left multiplication. Show that this action is transitive. By labelling the elements of S_3, gives an explicit transitive subgroup of S_6 isomorphic to S_3.

   (c) Using the Sylow theorems, show that S_5 has 6 Sylow 5-subgroups. Explain how this induces an embedding of S_5 into S_6 as a transitive subgroup.

(2) Find the Galois groups of each of the following polynomials

   (a) $x^3 - 3 \in \mathbf{Q}[x]$

   (b) $(x^2 - 2)(x^2 - 3) \in \mathbf{Q}[x]$