Assignment 2 – Part 1 – Math 612

(1) Suppose $\mathcal{C}$ is a category with a zero object (so that one can define kernels and cokernels). Show that every kernel is a monomorphism and every cokernel is an epimorphism.

(2) Prove the following statements given in class.

(a) Suppose $\mathcal{C}$ is an additive category and $f : A \to B$ is a morphism in $\mathcal{C}$ whose kernel exists. Show that f is monic if and only if its kernel is 0 (i.e. its kernel is the equivalence class of the zero morphism $0 : 0 \to A$.

(b) Suppose $\mathcal{C}$ is an additive category and $g : B \to C$ is a morphism in $\mathcal{C}$ whose kernel exists. Show that g is epic if and only if its cokernel is 0 (i.e. its cokernel is the equivalence class of the zero morphism $0 : C \to 0$.

(c) Show that, in an abelian category, kernels are monic and cokernels are epic.

(d) Show that if f is a morphism in an abelian category, then $\mathrm{im}(f) = \ker(\mathrm{coker}(f))$.