

Assignment – Part 1 – Math 612

- (1) Let R be a commutative ring. Recall from class that if M is a left (or right) R -module, equipped with its standard bimodule structure, and N is a left R -module, then $M \otimes_R N$ is naturally a left R -module via

$$r \cdot (m \otimes n) = (rm) \otimes n.$$

Also recall that if M, N , and X are left R -modules, then a function $\psi : M \times N \rightarrow X$ is called R -bilinear if it is R -biadditive and

$$r\psi(m, n) = \psi(rm, n) = \psi(m, rn)$$

for all $r \in R$, $m \in M$, and $n \in N$.

- (a) Show that when R is commutative, the natural map $\psi_{\text{univ}} : M \times N \rightarrow M \otimes_R N$ is R -linear.
- (b) Show that when R is commutative, $M \otimes_R N$ satisfies the following universal property: for every R -module X and every R -bilinear map $\psi : M \times N \rightarrow X$, there is a unique R -linear map φ such that the diagram

$$\begin{array}{ccc} M \times N & \xrightarrow{\psi_{\text{univ}}} & M \otimes_R N \\ & \searrow \psi & \downarrow \exists! \varphi \\ & & X \end{array}$$

commutes.

- (2) Let R be a ring, let $f : A \rightarrow A'$ and $f' : A' \rightarrow A''$ be homomorphisms of right R -modules, and let $g : B \rightarrow B'$ and $g' : B' \rightarrow B''$ be homomorphisms of left R -modules. Show that $(f' \otimes g') \circ (f \otimes g) = (f' \circ f) \otimes (g' \circ g)$.