

Assignment – Part 1 – Math 612

- (1) Let $\varphi : R \rightarrow S$ be a homomorphism of rings and let \mathcal{F} be the forgetful functor from $S\text{-Mod}$ to $R\text{-Mod}$. We showed in class that \mathcal{F} has a left adjoint, namely $S \otimes_R -$. Show that $\text{Hom}_R(S, -)$ is a right adjoint to \mathcal{F} . (Remark: thus \mathcal{F} has both a left and a right adjoint; the left adjoint is sometimes called *induction (from R to S)* while the right adjoint is sometimes called *coinduction (from R to S)*. They are sometimes isomorphic, but other times not.)

- (2) Let G be a group and let $G\text{-Set}$ be the category of G -sets (i.e. the objects are sets equipped with a left action of G and the morphisms are the G -equivariant functions between the sets). Let $\mathcal{F} : G\text{-Set} \rightarrow \mathbf{Set}$ be the forgetful functor.
 - (a) Given a set X , define $\text{Ind}^G(X)$ to be the set $G \times X$ with the action $\gamma \cdot (g, x) := (\gamma g, x)$. Show that Ind^G is a left adjoint to \mathcal{F} .
 - (b) Given a set Y , define $\text{coInd}^G(Y)$ to be the set of functions $\text{Hom}_{\mathbf{Set}}(G, Y)$ with the action $(\gamma \cdot \psi)(g) := \psi(g\gamma)$. Show that coInd^G is a right adjoint to \mathcal{F} .