**Assignment 8 – Part 1 – Math 612**

(1) Let $K/F$ be an algebraic field extension.

(a) Let $\{\alpha_i : i \in I\} \subseteq K$. Show that $F(\{\alpha_i : i \in I\}) = \{f((\alpha_i)_{i\in I}) : f \in F[I]\}$, i.e. every element of $F(\{\alpha_i : i \in I\})$ is simply a polynomial in the $\alpha_i$ with coefficients in $F$ (i.e. you don't need to take fractions of polynomials).

(b) Let $K_1$ and $K_2$ be two extensions of $F$ contained in $K$. Suppose $\{v_n : n \in N\}$ is a basis of $K_1$ (as an $F$-vector space) and $\{w_m : m \in M\}$ is a basis of $K_2$ (as an $F$-vector space). Show that the compositum is spanned by $\{v_n w_m : n \in N, m \in M\}$ (as an $F$-vector space). (Hint: one has that $K_1 = F(\{v_n : n \in N\})$ and similarly for $K_2$.)

(c) Conclude that $[K_1K_2 : F] \leq [K_1 : F][K_2 : F]$. When the $[K_i : F]$ are finite and relatively prime, show that equality holds.

(d) Suppose $f(x)$ and $g(x)$ are irreducible in $F[x]$. Let $\alpha$ be a root of $f(x)$. If the degrees of $f$ and $g$ are relatively prime, show that $g(x)$ is irreducible in $F(\alpha)[x]$.

(2) Suppose $K/F$ is an algebraic extension. Show that it is the compositum of all its finite sub extensions (i.e. of all $k \leq K$ such that $k$ is a finite extension of $F$).