

**Assignment 9 – All 1 part – Math 612**

**Due in class: Thursday, Apr. 11, 2019**

- (1) Let  $K$  and  $L$  be two finite extensions of a field  $F$  contained in some bigger field  $E$ .
  - (a) Show that  $[KL : F] \leq [K : F][L : F]$ .
  - (b) Show that if  $[K : F]$  and  $[L : F]$  are relatively prime, then  $[KL : F] = [K : F][L : F]$ .
- (2) Find the splitting field of each of the following polynomials.
  - (a)  $x^4 - 3 \in \mathbf{Q}[x]$
  - (b)  $x^2 - \sqrt{3} \in \mathbf{Q}(\sqrt{3})[x]$
  - (c)  $(x^2 - 2)(x^2 - 3) \in \mathbf{Q}[x]$
- (3) Suppose  $p$  is a prime and  $F$  is field of characteristic  $p$ . Show that every algebraic extension of  $F$  is separable if and only if every element of  $F$  has a  $p$ th root in  $F$ .