**Assignment 2 – Part 1 – Math 661**

(1) Some pure fields.

(a) Let $p$ be an odd prime such that $p^2 \nmid 2^{p-1}-1$. Let $\alpha = 2^{1/p} \in \mathbf{R}$ and let $K = \mathbf{Q}(\alpha)$. Show that

$$\Delta(K) = (-1)^{(p-1)/2} p^p 2^{p-1}$$

and

$$\mathcal{O}_K = \mathbf{Z}[2^{1/p}].$$

(Hint: look at what we did in class for $p = 3$.)

(b) Let $p$ be an odd prime, let $\alpha = p^{1/p} \in \mathbf{R}$, and let $K = \mathbf{Q}(\alpha)$. Show that $\Delta(K) = (-1)^{(p-1)/2} p^{2p-1}$ and $\mathcal{O}_K = \mathbf{Z}[p^{1/p}]$.

(2) As mentioned in class Theorem 4.26 of Narkiewicz says: Suppose $K_1, K_2$ are two number fields of degree $n_1, n_2$ with relatively prime discriminants. If $\alpha_1, \ldots, \alpha_{n_1}$ is an integral basis of $K_1$ and $\beta_1, \ldots, \beta_{n_2}$ is an integral basis of $K_2$, then $\{\alpha_i \beta_j\}$ is an integral basis of $K_1 K_2$ and

$$\Delta(K_1 K_2) = \Delta(K_1)^{n_2} \Delta(K_2)^{n_1}.$$

Use this to help you do the following.

(a) Let $p, q$ be distinct odd primes that are not both congruent to 3 (mod 4). Find the discriminant and an integral basis of $K = \mathbf{Q}(\sqrt{p}, \sqrt{q})$.

(b) Find the discriminant and an integral basis of $K = \mathbf{Q}(\sqrt[3]{2}, \zeta_5)$ (where $\zeta_5 = e^{2\pi i/5}$).