Math 3430
Series Solutions Worksheet
October 21, 2020

1. Find the recurrence relations that determine the coefficients of the series solutions centered at $t = 0$ for the following ODEs:

   (a) $y'' - y' - y = 3t^2$

   (b) $(1 + t^4)y'' - ty' + 3t^2y = 0$

2. Find the general solution of the following ODEs:

   (a) $y'' + 2ty' + 2y = 0$

   (b) $y'' - 2ty' + 2my = 0$ where m is a positive integer.

3. In this exercise we'll find the general solution to Bessel's differential equation:

$$t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} + (t^2 - \alpha^2)y = 0, \tag{1}$$

   where $\alpha > 0$.

   (a) Attempt to find a series solution to Bessel's ODE with the usual approach, i.e. assume $y(t) = \sum_{n=0}^{\infty} a_n t^n$. What happens in this case? Does this violate our theorem about solving ODEs via power series? Why or why not?

   (b) To find a series solution, you will have to modify the series a bit. Let $y(t) = \sum_{n=0}^{\infty} a_n t^{n+\lambda}$ where λ is any nonzero real number. Find the recurrence relations by making this assumption about the form of the solution. Make sure to pay special attention to the t^λ and $t^{\lambda+1}$ terms.

(c) Solve the recurrence relation you found in the previous question.